5.1 Integer Exponents and Scientific Notation
What You Will Learn

- Use the rules of exponents to simplify expressions.
- Rewrite exponential expressions involving negative and zero exponents.
- Write very large and very small numbers in scientific notation.
Rules of Exponents
Rules of Exponents

Let $m$ and $n$ be positive integers, and let $a$ and $b$ represent real numbers, variables, or algebraic expressions.

**Rule**

1. Product: $a^m \cdot a^n = a^{m+n}$

2. Product-to-Power: $(ab)^m = a^m \cdot b^m$

3. Power-to-Power: $(a^m)^n = a^{mn}$

4. Quotient: $\frac{a^m}{a^n} = a^{m-n}, m > n, a \neq 0$

5. Quotient-to-Power: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

**Example**

- $x^5(x^4) = x^{5+4} = x^9$
- $(2x)^3 = 2^3(x^3) = 8x^3$
- $(x^2)^3 = x^2 \cdot 3 = x^6$
- $\frac{x^5}{x^3} = x^{5-3} = x^2, x \neq 0$
- $\left(\frac{x}{4}\right)^2 = \frac{x^2}{4^2} = \frac{x^2}{16}$
Example 1 – Using Rules of Exponents

a. \((x^2y^4)(3x) = 3(x^2 \cdot x)(y^4) = 3(x^{2+1})(y^4) = 3x^3y^4\)

b. \(-2(y^2)^3 = (-2)(y^{2 \cdot 3}) = -2y^6\)

c. \((-2y^2)^3 = (-2)^3(y^2)^3 = -8(y^{2 \cdot 3}) = -8y^6\)

d. \((3x^2)(-5x)^3 = 3(-5)^3(x^2 \cdot x^3) = 3(-125)(x^{2+3}) = -375x^5\)

e. \(\frac{14a^5b^3}{7a^2b^2} = 2(a^{5-2})(b^{3-2}) = 2a^3b\)
Example 1 – Using Rules of Exponents

f. \[ \left( \frac{x^2}{2y} \right)^3 = \frac{(x^2)^3}{(2y)^3} = \frac{x^2 \cdot 3}{2^3 y^3} = \frac{x^6}{8y^3} \]

g. \[ \frac{x^n y^{3n}}{x^2 y^4} = x^n - 2y^{3n} - 4 \]
Integer Exponents
Definitions of Zero Exponents and Negative Exponents

Let $a$ and $b$ be real numbers such that $a \neq 0$ and $b \neq 0$, and let $m$ be an integer.

1. $a^0 = 1$
2. $a^{-m} = \frac{1}{a^m}$
3. $(\frac{a}{b})^{-m} = \left(\frac{b}{a}\right)^m$
Example 2 – Using Rules of Exponents

a. $3^0 = 1$

b. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

c. $\left(\frac{3}{4}\right)^{-1} = \left(\frac{4}{3}\right)^1 = \frac{4}{3}$
# Integer Exponents

## Summary of Rules of Exponents

Let \( m \) and \( n \) be integers, and let \( a \) and \( b \) represent real numbers, variables, or algebraic expressions. (All denominators and bases are nonzero.)

**Product and Quotient Rules**

<table>
<thead>
<tr>
<th>Rule</th>
<th>Expression</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( a^m \cdot a^n = a^{m+n} )</td>
<td>( x^4(x^3) = x^{4+3} = x^7 )</td>
<td></td>
</tr>
<tr>
<td>2. ( \frac{a^m}{a^n} = a^{m-n} )</td>
<td>( \frac{x^3}{x} = x^{3-1} = x^2 )</td>
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</table>

**Power Rules**

<table>
<thead>
<tr>
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<th>Example</th>
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</thead>
<tbody>
<tr>
<td>3. ( (ab)^m = a^m \cdot b^m )</td>
<td>( (3x)^2 = 3^2(x^2) = 9x^2 )</td>
<td></td>
</tr>
<tr>
<td>4. ( (a^m)^n = a^{mn} )</td>
<td>( (x^3)^3 = x^3 \cdot 3 = x^9 )</td>
<td></td>
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<tr>
<td>5. ( \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} )</td>
<td>( \left(\frac{x}{3}\right)^2 = \frac{x^2}{3^2} = \frac{x^2}{9} )</td>
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**Zero and Negative Exponent Rules**

<table>
<thead>
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<th>Expression</th>
<th>Example</th>
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</thead>
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<tr>
<td>6. ( a^0 = 1 )</td>
<td>( (x^2 + 1)^0 = 1 )</td>
<td></td>
</tr>
<tr>
<td>7. ( a^{-m} = \frac{1}{a^m} )</td>
<td>( x^{-2} = \frac{1}{x^2} )</td>
<td></td>
</tr>
<tr>
<td>8. ( \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m )</td>
<td>( \left(\frac{x}{3}\right)^{-2} = \left(\frac{3}{x}\right)^2 = \frac{3^2}{x^2} = \frac{9}{x^2} )</td>
<td></td>
</tr>
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</table>
Scientific Notation
Exponents provide an efficient way of writing and computing with very large and very small numbers. For instance, a drop of water contains more than 33 billion molecules—that is, 33 followed by 18 zeros. It is convenient to write such numbers in **scientific notation**.

This notation has the form $c \times 10^n$, where $1 \leq c < 10$ and $n$ is an integer. So, the number of molecules in a drop of water can be written in scientific notation as follows.

$$33,000,000,000,000,000,000 = 3.3 \times 10^{19}$$

19 places
Example 7 – Writing in Scientific Notation

a. \(0.0000684 = 6.84 \times 10^{-5}\)  \(\text{Small number} \rightarrow \text{negative exponent}\)
   \(\text{Five places}\)

b. \(937,200,000.0 = 9.372 \times 10^{8}\)  \(\text{Large number} \rightarrow \text{positive exponent}\)
   \(\text{Eight places}\)
Homework:

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#’s 79 – 99 odd