Section 5.4

Logarithmic Functions

DEFINITION

The **logarithmic function to the base *a***, where $a > 0$ and $a \neq 1$, is denoted by $y = \log_a x$ (read as "y is the logarithm to the base a of x") and is defined by

$$y = \log_a x \quad \text{if and only if} \quad x = a^y$$

The domain of the logarithmic function $y = \log_a x$ is $x > 0$.

EXAMPLE

Relating Logarithms to Exponents

(a) If $y = \log_3 x$, then $x = 3^y$.

For example, $4 = \log_3 81$ is equivalent to $81 = 3^4$.

(b) If $y = \log_5 x$, then $x = 5^y$.

For example, $-1 = \log_5\left(\frac{1}{5}\right)$ is equivalent to $\frac{1}{5} = 5^{-1}$.

EXAMPLE

Changing Exponential Statements to Logarithmic Statements

Change each exponential expression to an equivalent expression involving a logarithm.

(a) $5^8 = t$ (b) $x^{-2} = 12$ (c) $e^x = 10$

$\log_5 t = 8$ $\log_x 12 = -2$ $\log_e 10 = x$

EXAMPLE

Changing Logarithmic Statements to Exponential Statements

Change each logarithmic expression to an equivalent expression involving an exponent.

(a) $\log_2 21 = y$ (b) $\log_z 12 = 6$ (c) $\log_2 10 = a$

$2^y = 21$ $z^6 = 12$ $2^a = 10$

EXAMPLE

Finding the Exact Value of a Logarithmic Expression

$(a)\ \log_3 81 = x$ $\quad (b)\ \log_2 \frac{1}{8} = x$

$3^x = 81$ $\quad 2^x = \frac{1}{8}$ $\quad 2^x = 8$

$x = 4$ $\quad x = -3$ $\quad x = 3$

Domain of the logarithmic function = Range of the exponential function = $(0, \infty)$

Range of the logarithmic function = Domain of the exponential function = $(-\infty, \infty)$

$y = \log_a x$ (defining equation: $x = a^y$)

Domain: $0 < x < \infty$ Range: $-\infty < y < \infty$

EXAMPLE

Finding the Domain of a Logarithmic Function

Find the domain of each logarithmic function.

$(a)\ f(x) = \log_3(x-2)$ $(b)\ F(x) = \log_2\left(\frac{x+3}{x-1}\right)$

$(c)\ h(x) = \log_2|x-1|$ $(d)\ g(x) = \log_{\frac{1}{2}} x^2$

Natural Logarithm Function

$$y = \ln x \quad \text{if and only if} \quad x = e^y$$

$\log_e = \ln$

ln is natural log

Base 10

$$y = \log x \quad \text{if and only if} \quad x = 10^y$$

$\log = \log_{10}$

EXAMPLE

Solving a Logarithmic Equation

Solve: $(a)\ \log_2(2x+1)=3$ $(b)\ \log_x 343=3$

$2^3 = 2x+1$

$8 = 2x+1$

$7 = 2x$

$x = 7/2$

$x^3 = 343$

$x = 7$

EXAMPLE

Using Logarithms to Solve Exponential Equations

Solve: $2e^{3x} = 6$

$e^{3x} = 3$

$\log_e 3 = 3x$

$\ln 3 = 3x$

$x = .36$

EXAMPLE Alcohol and Driving

The blood alcohol concentration (BAC) is the amount of alcohol in a person's bloodstream. A BAC of 0.04% means that a person has 4 parts alcohol per 10,000 parts blood in the body. Relative risk is defined as the likelihood of one event occurring divided by the likelihood of a second event occurring. For example, if an individual with a BAC of 0.02% is 1.4 times as likely to have a car accident as an individual who has not been drinking, the relative risk of an accident with a BAC of 0.02% is 1.4. Recent medical research suggests that the relative risk R of having an accident while driving a car can be modeled by the equation

$$R = e^{kx}$$

where x is the percent of concentration of alcohol in the bloodstream and k is a constant.

(a) Research indicates that the relative risk of a person having an accident with a BAC of 0.02% is 1.4. Find the constant k in the equation.

(b) Using this value of k, what is the relative risk if the concentration is 0.17%?

(c) Using this same value of k, what BAC corresponds to a relative risk of 100?

(d) If the law asserts that anyone with a relative risk of 5 or more should not have driving privileges, at what concentration of alcohol in the bloodstream should a driver be arrested and charged with a DUI (driving under the influence)?

Homework:

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#'s 9 - 53 every other odd, 87 - 93 odd, 117