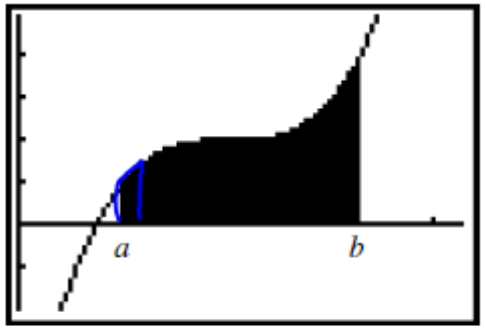
Section 4.2B/4.3A/4.6A – Riemann Sums

Objectives:

1. Understand the concept of area.
2. Approximate the area of a plane region.
3. Understand the definition of a Riemann sum.

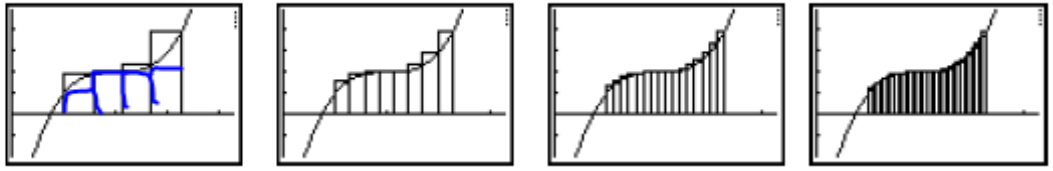


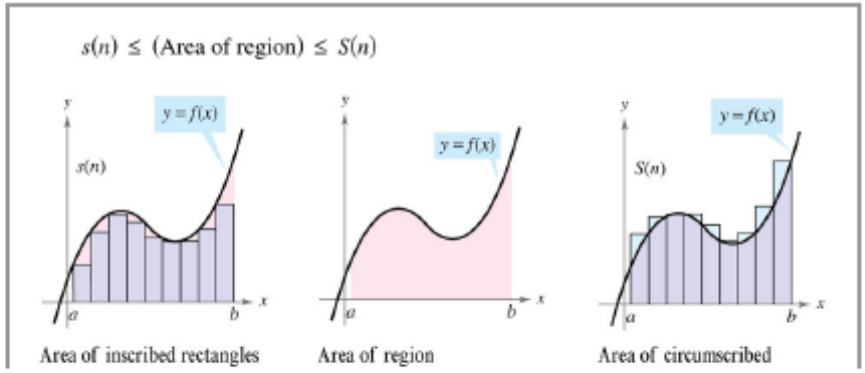
I. The Area of a Plane Region

A. Recall we there were two classic problems in calculus – the tangent problem and the area problem.
We have discussed the tangent problem so now we will begin our investigation of the area problem.

B. Method:

1. Break region into rectangles or trapezoids (of equal size, if possible).
2. Find the area of each region.
3. Find total area.

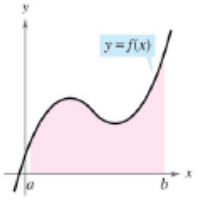




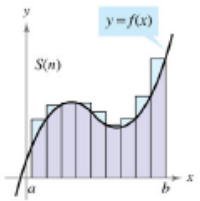
By increasing the number of rectangles you can obtain closer and closer approximations.

$s(n) \le$ (Area of region) $\le S(n)$

Area of inscribed rectangles is less than area of region.



Area of region



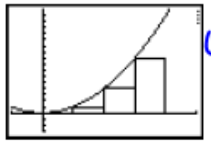
Area of circumscribed rectangles is greater than area of region.

By increasing the number of rectangles you can obtain closer and closer approximations.

Σ

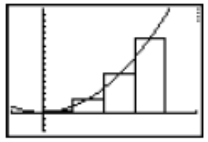
C. Four Types
Rectangular Methods

1. Left Sided Rectangles

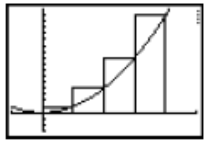


(under)

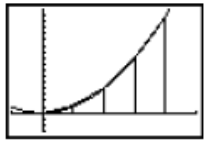
2. Middle Rectangles



3. Right Sided Rectangles



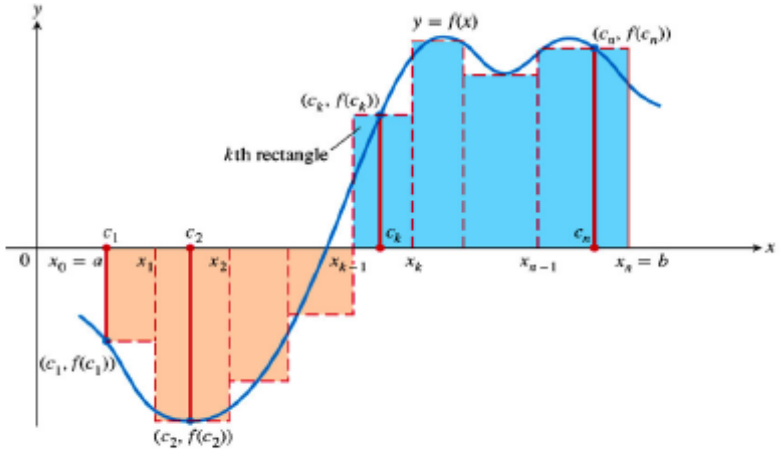
Trapezoid Method



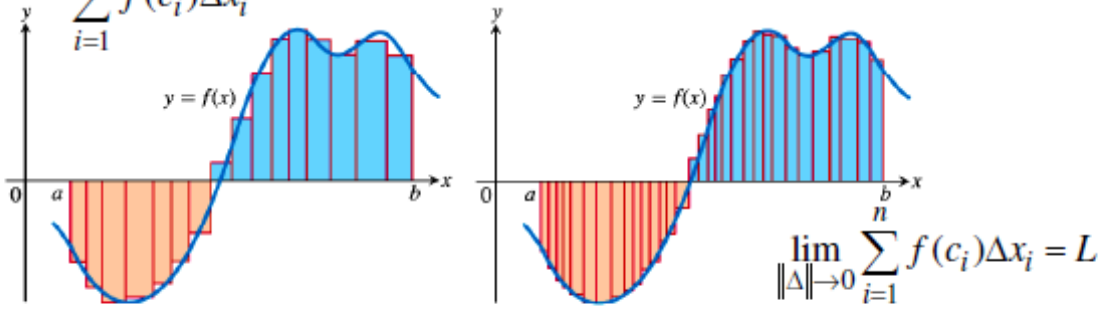
Do Calculator Lab #10

II. Riemann Sums

$$R_n = f(c_1)\Delta x_1 + f(c_2)\Delta x_2 + \ldots + f(c_n)\Delta x_n = \sum_{i=1}^{n} f(c_i)\Delta x_i$$



$$\sum_{i=1}^{n} f(c_i)\Delta x_i$$



$$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = L$$

III. The Definite Integral

A. $$I = \int_a^b f(x)dx = \lim_{\|\Delta\| \to 0} \sum_{k=1}^{n} f(c_i)\Delta x_i$$

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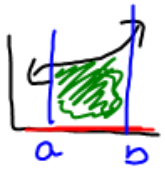


B. Definite Integral – If f is Continuous and non negative on the closed interval $[a,b]$, then the Area of the region bounded by the graph of f, the x-axis and the vertical lines $x = a$ and $x = b$ is given by

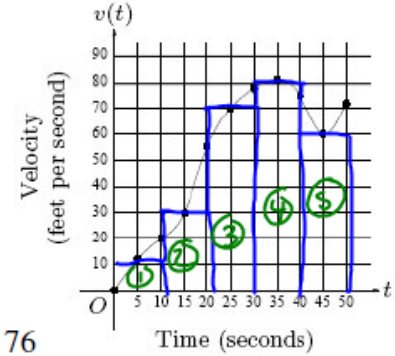
Area = $\int_a^b f(x)dx$

C. Example – Approximate $\int_0^{50} v(t)dt$ with a Riemann Sum, using the midpoints of five subintervals of equal length. Use correct units; explain the meaning of the integral.

$\Delta x = \frac{b-a}{n}$ ← # of rect.

$\Delta x = \frac{50-0}{5} = 10$

midpts → $\frac{\Delta x}{2} = 5$



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

$\sum_{i=1}^{5} v(t)(\Delta x) =$

Homework:
p. 268 – 29, 31, 33, 35, 42, 43, 73, 76
p. 280 – 9-12, 53, 54; and p. 316 – 3, 7, 52 (Trapezoid Rule Only)

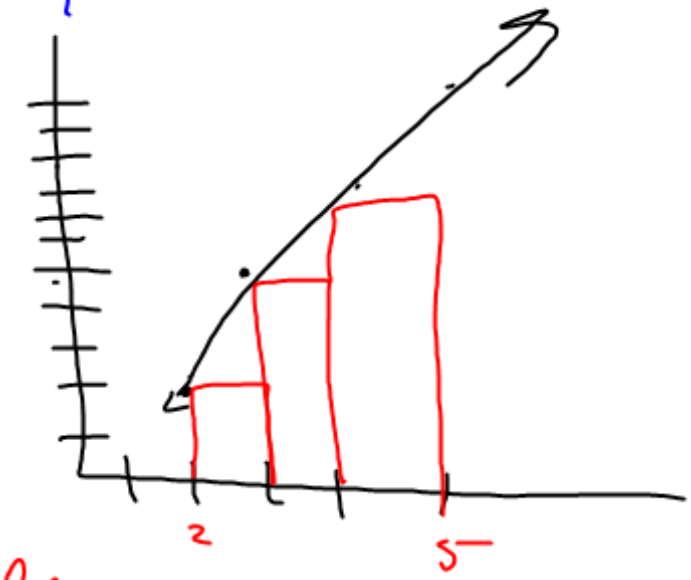
$10(12) + 10(30) + 10(70) + 10(81) + 10(60)$
① ② ③ ④ ⑤

$= 2530$

like 47

$y = 3x - 4$

$[2, 5]$ (a, b)



$1(2) + 1(5) + 1(8) = 15$

left:

$$\frac{5-2}{3} = 1$$

4.2B

Pg 268

23 -30 no + 24, 26

47, 49.57

No limit - Process