3.9 Differentials

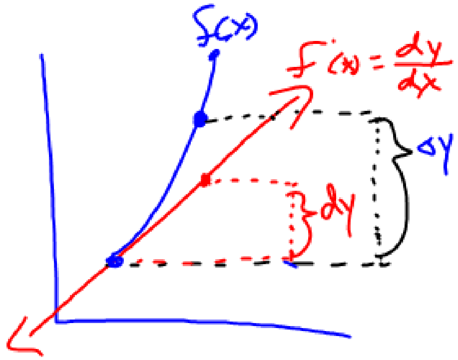
Pg 236

If $y = f(x)$, The Differential is $dy = f'(x)dx$

$y = f(x)$

$\frac{dy}{dx} = f'(x)$

$dy = f'(x)dx$



✓Think of the Differentials:

dy and *dx* as small changes in *y* and *x*.

Ex 1. Determine the change in the function $y = x^{2/3}$ when x decreases from 8 to 7.8.

$$y = x^{\frac{2}{3}}$$

$$\frac{dy}{dx} = \frac{2}{3}x^{-1/3}$$

so $dy = \frac{2}{3x^{1/3}}dx$ (Differential)

Know ① $x = 8$
$dx = -.2$

$$dy = \frac{2}{3\sqrt[3]{8}}(-.2)$$

$$dy = -.0\overline{6}$$

or $-1/15$

Find the differential (dy) of the given function: $y = \sqrt{9 - x^2}$

$$\frac{dy}{dx} = \frac{1}{2}(9 - x^2)^{-1/2}(-2x)$$

$$\boxed{dy = \frac{-x}{\sqrt{9 - x^2}}\,dx}$$

Use linear Approximation (tangent lines) to find the value of t $\sqrt[3]{8.5}$ expression. Compare your answer with that of a calculator.

$$\sqrt[3]{8.5}$$

① $\sqrt[3]{x} = y \quad x = 8 \quad y = 2$

② $y' = \frac{1}{3}(x)^{-2/3}$

$y' = \frac{1}{3\sqrt[3]{x^2}} = \frac{1}{3 \cdot 4} = \frac{1}{12}$ (slope)

③ $y - 2 = \frac{1}{12}(x - 8)$

$y - 2 = \frac{1}{12}(8.5 - 8)$

$y = 2 + \frac{1}{12}(.5)$

$y \approx 2.04166$

Actual: $\sqrt[3]{8.5} \approx 2.040827$

Ex. 3 Same problem different way to solve: Determine an approximation for $\sqrt{15.5}$

Compare dy with Δy

$\sqrt[3]{8.5} \rightarrow x = 8$
$dx = .5$

$dy = f'(x)dx$

$dy = \frac{1}{3}(x)^{-2/3}dx$

$dy = \frac{1}{3\sqrt[3]{x^2}}dx$

$dy = \frac{1}{12}(.5)$

$dy = .04166$

2.04166

$\sqrt[3]{8} + \sqrt[3]{.5} =$

$2 + .04166 = 2.04166$

$\Delta y = f(x \pm \Delta x)$

↑ Actual

$\sqrt{15.5}$

$\sqrt{x}$ $\quad x = 16$ $\quad dx = -.5$

$y = 4$

$dy = \frac{1}{2} x^{-1/2} dx$

$dy = \frac{1}{2\sqrt{x}} dx$

$dy = \frac{1}{2\sqrt{16}} (-.5)$

$dy = \frac{1}{8} \cdot \frac{-1}{2}$

$dy = -1/16$

$y \pm dy$

$\sqrt{15.5} \approx 4 - \frac{1}{16}$

≈ 3.9375

Actual

$\sqrt{15.5} \approx 3.937$

The measured radius of a ball bearing is 0.7 inch. If the measurement is correct within 0.01 inch, estimate the propagated
error int he volume V of the ball bearing.

2. Find the relative error and percent of error.

$V = \frac{4}{3}\pi r^3$

$r = .7$

Since $\Delta V \approx dV$ use dV

$dr = \pm .01$

Relative Error = $\dfrac{\text{differential}}{\text{actual}}$

$dV = 4\pi r^2 dr$

$dV = 4\pi(.7)^2(\pm .01)$

$\approx \pm .06158$ cubic inches (Propagated error)

take $\dfrac{dV}{V} = \dfrac{\pm .06158}{\frac{4}{3}\pi(.7)^3} = \dfrac{\pm .06158}{1.43676} \approx \pm .04286$ Relative Error

Percent of Error: $\pm 4.29\%$

Ex 5   If the radius of a circle is measured to be 12cm with a possible error of $\pm$ .03cm, find the percent error in calculating the Area of the circle.

HW: pg 240 27, 30, 31

$\sqrt{50}$ $\sqrt{98}$ $\sqrt[3]{127}$