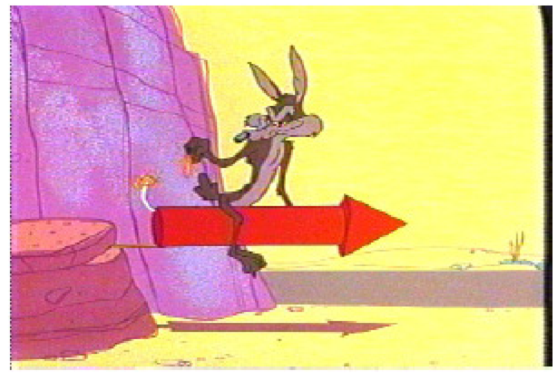
# Optimization [also called min/max problems]

A warm-up problem from http://chaoticgolf.com
[Mr. Leckie's awesome website]

*Example*: Wile E. is after Road Runner again! This time he's got it figured out. Sitting on his ACME rocket he hides behind a hill anxiously awaiting the arrival that "beeping" bird. In his excitement to light the rocket he tips the rocket up. Instead of thrusting himself parallel to the ground where he can catch the Road Runner, he sends himself widely into the air following a path given by the position function

$$s(t) = -.00086x^4 + .067x^3 - 1.67x^2 + 14.77x.$$

How high does Wile E. go, and when does he reach that height?



Find the maximum height [using Calculus]

$$s(t) = -0.00086x^4 + 0.067x^3 - 1.67x^2 + 14.77x$$

$$s'(t) = -.00344x^3 + .201x^2 - 3.34x + 14.77$$

$s'(t) = 0$ at $t \approx 18.515$

$t \approx 7.058$

$t \approx 32.857$

At $t \approx 7.058$ and $t \approx 32.857$ $s'(t)$ changes from positive to negative values ~~Hence~~ $s(t)$ has rel max at $t \approx 7.058$ and $t \approx 32.857$

$s(7.058) \approx 42.478$ m

$s(32.857) \approx 56.680$ m

max height

And now on to optimization! [Don't confuse this with related rates]
See Guidelines on page 219

Find two positive numbers such that the product is 192 and the sum is a minimum.

Product $x\,y = 192$
Sum: $sum = x + y$ which we want to minimize

Primary equation: $x + y$ = sum Let's call the sum $s(x)$
Secondary equation: $xy = 192$

Rewrite the primary equation using information from the secondary equation so that the primary equation only has one variable.

Let sum be $s(x)$

$x\,y = 192$

$y = \dfrac{192}{x}$

Now, $s(x) = x + \dfrac{192}{x}$

Domain: (0,192]
Now use calculus to find the minimum value of $x$

$s'(x) = 1 - \frac{192}{x^2}$

Note: $s'(x)$ is undefined at $x = 0$ but $x = 0$ is NOT in our domain

Let $s'(x) = 0$

$0 = 1 - \frac{192}{x^2}$

$\frac{192}{x^2} = 1$

$192 = x^2$

$\sqrt{192} = x$

$y = \frac{192}{x}$

$y = \frac{192}{\sqrt{192}} = \sqrt{192}$

$(0, \sqrt{192})$ $(\sqrt{192}, 192)$

$s'(x) < 0$ $s'(x) > 0$

at $x = \sqrt{192}$

$s'(x)$ CHANGES FROM neg TO POS VALUES

Hence $s(x)$ has a rel min at $x = \sqrt{192}$

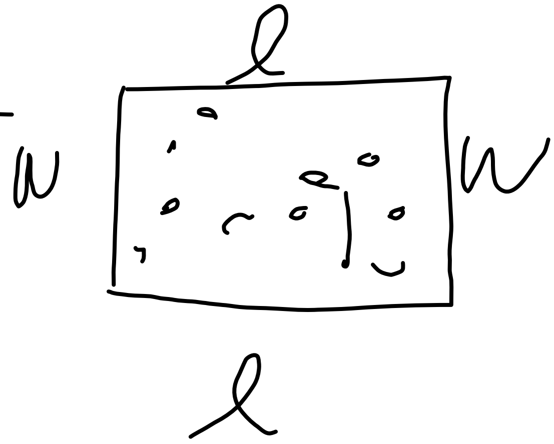
OUR MIN SUM

$\sqrt{192} + \sqrt{192}$

$= 2\sqrt{192}$

Example 2: Find the length and width of a rectangle that has a perimeter of 64 feet and a maximum area.

$P = 2l + 2w$ SECONDARY
$A = lw$ PRIMARY




$64 = 2l + 2w$

Re-write: $l = 32 - w$

$A(w) = w(32 - w)$ DOMAIN: $(0, 32)$

$A(w) = 32w - w^2$

$A'(w) = 32 - 2W$ $0 = 32 - 2W$

Let $A'(w) = 0$

Our critical value is $W = 16$

$(0, 16)$ $(16, 32)$

$A'(w) > 0$ $A'(w) < 0$

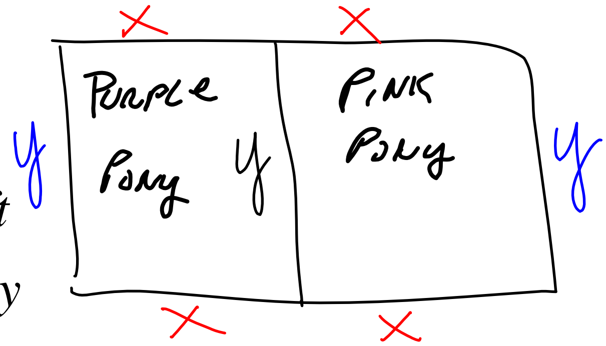
at $w = 16$ $A'(w)$ changes from pos to neg values Hence $A(w)$ has a rel max at $w = 16$

$w = 16$ $l = 16$

max area = 256 sq ft

Let's look at #20 on page 224

It might help to draw a picture first to see what variables we will need. Here is my awesome rendering




$P = 200\ ft$

$P = 4x + 3y$

$A = 2xy$ PRIMARY

$200 = 4x + 3y$ SECONDARY

Re-write: $200 - 4x = 3y$

$$\frac{200-4x}{3} = y$$

Now to find the maximum area

Domain: $(0, 50)$

$$A(x) = (2x)\left(\frac{200-4x}{3}\right)$$

$$A(x) = \frac{8}{3}\left(50x - x^2\right)$$

$$A'(x) = \frac{8}{3}(50-2x)$$

$$0 = \frac{8}{3}(50-2x)$$

Let $A'(x) = 0$ to find any critical values

Our only c.v. is $x = 25$

$(0, 25)$ $\quad$ $(25, 50)$

$A'(x) > 0$ $\quad$ $A'(x) < 0$

at $X=25$ $A'(x)$ changes from pos to neg value hence $A(x)$ has a rel max at $X=25$.

$$X = 25 \text{ ft}$$

$$y = \frac{200-4x}{3}$$

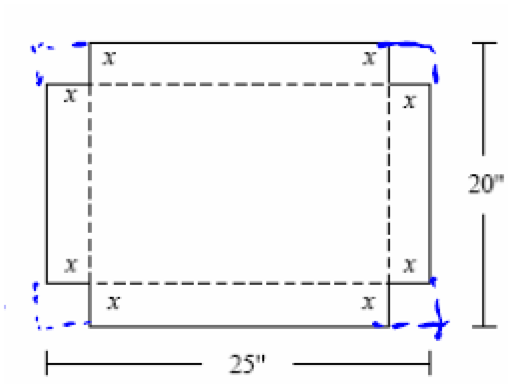
$$y = \frac{100}{3} \text{ ft}$$

$$A(25) = 2(25)\left(\frac{100}{3}\right)$$

$$= \frac{5000}{3} \text{ sq ft}$$

HOMEWORK FOR TUESDAY = #8 AND 9 ON THE CH3 FREE RESPONSE HANDOUT [IF YOU NEED ONE, YOU CAN PRINT IT OUT FROM OUR WEBSITE]

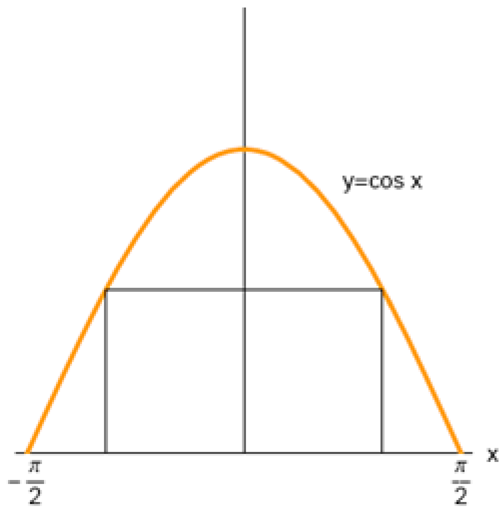
Let's try this problem from http://chaoticgolf.com
[Mr. Leckie's awesome website]
An open-top box is to be made by cutting congruent squares of side length $x$ from the corners of a 20 – by 25-inch sheet of tin and bending up the sides. How large should the squares be to make the box hold as much as possible?




Here's a classic optimization problem:

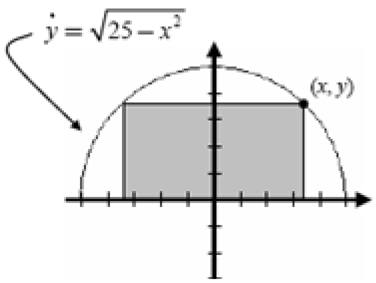
From:
http://www.frapanthers.com/teachers/zab/APCalculusInaNutshell/ApplicationsDerivatives2004.pdf

A rectangle is to be inscribed under one arch of the cosine curve.  What is the largest area the rectangle can have and what dimensions give that area?



You try this problem from http://chaoticgolf.com
[Mr. Leckie's awesome website]

A rectangle is bounded by the x-axis the semi-circle $y=\sqrt{25-x^2}$ . What length and width should the rectangle have so that its area is a maximum?

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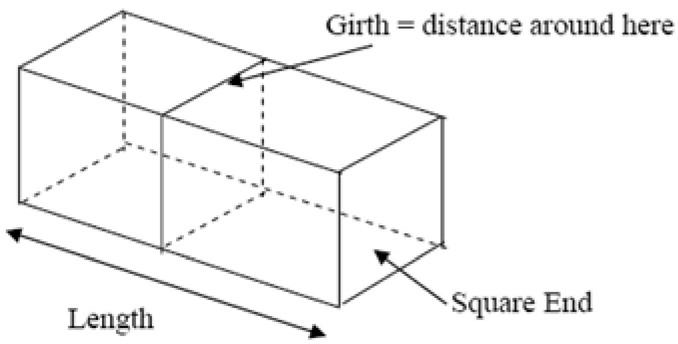
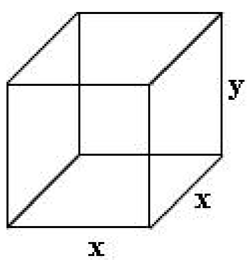
And now, for my favorite of all optimization problems:
[page 225 #33]




$V = l\,w\,h$ this is way too many variables

Another classic problem:
From:
http://www.mathematicshelpcentral.com/lecture_notes/calculus_1_folder/min-max_and_optimization_problems.htm

A box with a square base with NO top has a surface area of 108 square feet. Find the dimensions that will maximize the volume. [You must use Calculus!]




$$SA = x^2 + 4xy$$
$$108 = x^2 + 4xy$$

Homework: page 223-225 #5, 11, 19, 27, 34 [for #34 the V = $\pi r^{2} h$ and the girth would be $= 2\pi r$]
Use your handy-dandy TI to find the critical values and you must how all of your steps like we just did in class.
Remember to be mindful of domain issues.