5.2 Log Integration

Log Rule for Integration

1. $\int \frac{1}{x}\,dx = \ln|x| + C$

$\frac{d}{dx}[\ln x] = \frac{1}{x}$

2. $\int \frac{1}{u}\,du = \ln|u| + C$

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Ex1 $\int \frac{2}{x}\,dx = 2\int \frac{1}{x}\,dx$

$2\ln x + C$

$\ln x^2 + C$

Ex2 $\int \frac{1}{4x-1}\,dx$

$u = 4x - 1$

$du = 4\,dx$

$\frac{1}{4}\int \frac{1}{u}\,du$

$\frac{1}{4}\ln u + C$

$\frac{1}{4}\ln|4x-1| + C$

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Ex 3 Finding the Area with Log Rules
Find the area bounded by the x-axis and line x = 3

$y = \frac{x}{x^2+1} \Rightarrow \int_0^3 \frac{x}{x^2+1}\,dx$

$u = x^2 + 1$
$du = 2x\,dx$

$\frac{1}{2}\int_0^3 \frac{2x}{x^2+1}\,dx$

$\frac{1}{2}\int_0^3 \frac{1}{u}\,du$

$\frac{1}{2}\ln u \Big]_0^3$

$\frac{1}{2}\ln x^2+1 \Big]_0^3$

$\frac{1}{2}[\ln 10 - \ln 1]$ ($\ln 1 = 0$)

$\frac{1}{2}\ln 10 \approx 1.151$

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Ex 4 Quotient Form

a. $\int \frac{3x^2+1}{x^3+x}\,dx$

$\ln|x^3 + x| + C$

b. $\int \frac{\sec^2 x}{\tan x}\,dx$

$\ln|\tan x| + C$

c. $\int \frac{x+1}{x^2+2x}\,dx$

$\frac{1}{2}\ln|x^2+2x| + C$

$u = x^2 + 2x$
$du = 2x + 2\,dx$

d. $\int \frac{1}{3x+2}\,dx$

$\frac{1}{3}\ln|3x+2| + C$

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Ex5) Long Division

$$\int \frac{x^2+x+1}{x^2+1}$$

$$x^2+1 \;\overline{)\; x^2+x+1}$$

$$1 + \frac{x}{x^2+1}$$

$$-x^2 \quad +1$$

$$x$$

$$\int 1 + \frac{x}{x^2+1}\,dx$$

$$x + \frac{1}{2}\ln|x^2+1| + C$$

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Ex6) Change of Variable

$$\int \frac{2x}{(x+1)^2}\,dx$$

$u = x+1 \Rightarrow x = u-1$

$du = dx$ still an x

$$\int \frac{2(u-1)}{u^2}\,du$$

$$\int \frac{2u-2}{u^2}\,du$$

$$\int \frac{2u}{u^2} - \frac{2}{u^2}\,du$$

$$\int \frac{2}{u} - 2u^{-2}\,du$$

$2\int \frac{1}{u}$ ←

$$2\ln|u| - \frac{2u^{-1}}{-1} + C$$

$$2\ln|x+1| + \frac{2}{x+1} + C$$

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Ex 7) u-Sub and log Rule

$\frac{dy}{dx} = \frac{1}{x \ln x}$

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HW

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