

5.2 Log Integration

Log Rule for Integration

$$1. \int \frac{1}{x} dx = \ln|x| + C$$

$$2. \int \frac{1}{u} du = \ln|u| + C$$

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Ex 1 $\int \frac{2}{x} dx =$

$$2 \int \frac{1}{x} dx = 2 \ln|x| + C$$

Ex 2 $\int \frac{1}{4x-1} dx$

$$u = 4x - 1$$

$$du = 4 dx$$

$$\rightarrow \frac{1}{4} \int \frac{4}{4x-1} dx$$

$$\frac{1}{4} \int \frac{1}{u} du$$

$$\frac{1}{4} \ln|u| + C$$

$$\frac{1}{4} \ln|4x-1| + C$$

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Ex 3 Finding the Area with Log Rules
Find the area bounded by the x-axis and line $x = 3$

$$y = \frac{x}{x^2+1} \Rightarrow \int_0^3 \frac{x}{x^2+1} dx$$

$$u = x^2 + 1 \quad \frac{1}{2} \int \frac{1}{u} du$$

$$du = 2x dx$$

$$\left. \frac{1}{2} \ln|u| \right]_0^3$$

$$\frac{1}{2} \ln|x^2+1| \Big|_0^3$$

$$\frac{1}{2} \ln|9| - \frac{1}{2} \ln|1|$$

$$\frac{1}{2} \ln|9| \quad \times \ln|1| = 0$$

$$\approx 1.51$$

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Ex 4 Quotient Form

a. $\int \frac{3x^2+1}{x^3+x} dx$

$\ln|x^3+x| + C$

b. $\int \frac{\sec^2 x}{\tan x} dx$

$\ln|\tan x| + C$

c. $\int \frac{x+1}{x^2+2x} dx$

$\frac{1}{2} \ln|x^2+2x| + C$

d. $\int \frac{1}{3x+2} dx$

$\frac{1}{3} \ln|3x+2| + C$

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Ex5 Long Division

$$\int \frac{x^2 + x + 1}{x^2 + 1}$$

$$x^2 + 1 \overline{) x^2 + x + 1} \quad + \frac{x}{x^2 + 1}$$

$$\underline{-x^2} \quad \quad \quad \underline{+1}$$

$$x$$

$$\int 1 + \frac{x}{x^2 + 1} dx = \int 1 dx + \int \frac{x}{x^2 + 1} dx$$

$$u = x^2 + 1$$

$$du = 2x dx$$

$$x + \frac{1}{2} \int \frac{1}{u} du$$

$$x + \frac{1}{2} \ln|x^2 + 1| + C$$

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Ex6 Change of Variable

$$\int \frac{2x}{(x+1)^2} dx$$

$$u = x + 1 \quad x = u - 1$$

$$du = dx$$

$$\int \frac{2(u-1)}{u^2} du$$

$$\frac{2}{u^2} - \frac{1}{u} \Rightarrow \int \frac{2u}{u^2} - \frac{2}{u^2} du$$

$$2 \int \frac{1}{u} du - 2 \int u^{-2} du$$

$$2 \ln|u| - 2 \cdot \frac{u^{-1}}{-1} + C$$

$$2 \ln|x+1| + \frac{2}{x+1} + C$$

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Ex 7) u-Sub and log Rule

$$\frac{dy}{dx} = \frac{1}{x \ln x}$$

$$u = \ln x$$

$$du = \frac{1}{x}$$

$$\int \frac{1}{x \ln x} dx$$

$$\int \frac{1}{u} du$$

$$\ln|u|$$

$$\ln|\ln x| + c$$

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HW
Pg 338

1-23 odd
study

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