# 4.5 U-Substitution

* Basically undoing Chain Rule!

$$\int f(g(x))\,g'(x)\,dx = F(g(x)) + C$$

If $u = g(x)$ then

$du = g'(x)\,dx$

$$\int f(u)\,du = F(u) + C$$

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Ex)

$$\int (x^2+1)^2\,(2x)\,dx$$

→ inside Der.✓ !!

Let $u = x^2 + 1$

$du = 2x\,dx$

$$\int u^2\,du = \frac{u^3}{3} + C$$

$$= \frac{(x^2+1)^3}{3} + C$$

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Ex 2 $\int 5\cos 5x\,dx$

Let $u = 5x$

$du = 5dx$

$\int \cos u\,du$

$= \sin u + C$

$= \sin 5x + C$

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Ex 3 $\int x(x^2+1)^2\,dx$

Let $u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2}\int 2x(x^2+1)^2\,dx$

$\frac{1}{2}\int u^2\,du$

$\frac{1}{2}\cdot\frac{u^3}{3} + C$

$= \frac{(x^2+1)^3}{6} + C$

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Ex 4) $\int \sqrt{2x-1}\,dx$

Let $u = 2x-1$

$du = 2dx$

$\frac{1}{2}\int u^{1/2}\,du$

$\frac{1}{2}\cdot\frac{2}{3}$

$= \frac{1}{2}\left(\frac{u^{3/2}}{\frac{3}{2}}\right)$

$= \frac{u^{3/2}}{3}$

$= \frac{(2x-1)^{3/2}}{3} + C$

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Ex 5) $\int \sin^2(3x)\cos 3x\,dx$

Let $u = \sin 3x$

$du = 3\cos 3x\,dx$

$\frac{1}{3}\int u^2\,du = \frac{1}{3}\cdot\frac{u^3}{3}$

$= \frac{(\sin 3x)^3}{9} + C$

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Ex 6

$$\int \frac{-4x}{(1-2x^2)^2}\,dx = \int \frac{du}{u^2}$$

Let $u = 1 - 2x^2$

$du = -4x\,dx$

$$= \int u^{-2}\,du$$

$$= \frac{u^{-1}}{-1}$$

$$= \frac{-1}{(1-2x^2)} + C$$

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7 - 33 odd

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$$\int \frac{x^3}{(1+x^4)^2}\,dx$$

$u = 1 + x^4$

$du = 4x^3\,dx$

$$\frac{1}{4}\int \frac{4x^3}{(1+x^4)^2}\,dx$$

$$\frac{1}{4}\int \frac{1}{u^2}\,du$$

$$\frac{1}{4}\int u^{-2}\,du$$

$$\frac{1}{4}\left(\frac{u^{-1}}{-1}\right) + C$$

$$-\frac{1}{4(1+x^4)} + C$$

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$$\int \frac{x}{\sqrt{1+x^2}}\,dx$$

$u = 1 + x^2$

$du = 2x\,dx$

$$\frac{1}{2}\int \frac{1}{u^{1/2}}\,du$$

$$\frac{1}{2}\int u^{-1/2}\,du$$

$$\frac{1}{2}\,\frac{u^{1/2}}{\frac{1}{2}} + C$$

$$\frac{1}{2}\cdot\frac{2}{1}u^{1/2} + C$$

$$\left(1+x^2\right)^{1/2} + C$$

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$$\int_{-1}^{1} x(1-3x^2)^4\,dx$$

$u = 1-3x^2$

$du = -6x\,dx$

$$-\frac{1}{6}\int u^4\,du$$

$$-\frac{1}{6}\cdot\frac{u^5}{5}$$

$$\left[\frac{-(1-3x^2)^5}{30}\right]_{-1}^{1}$$

$\frac{32}{30} - \frac{32}{30}$

$0$

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67 - 87 odds

Mar 3-11:01 AM